$\qquad$ Date $\qquad$

## Mathinks

6-2
STUDENT PACKET

## MATHLINKS: GRADE 6 STUDENT PACKET 2 WHOLE NUMBER APPLICATIONS

| 2.1 | Factors and Multiples <br> - Understand what it means to find factors and multiples of numbers. <br> - Review the definitions of prime and composite numbers. | 1 |
| :---: | :---: | :---: |
| 2.2 | GCF and LCM <br> - Find the greatest common factor of two natural numbers. <br> - Find the least common multiple of two natural numbers. <br> - Use the greatest common factor and least common multiple to solve problems. | 10 |
| 2.3 | Numerical Expressions <br> - Apply the concept of greatest common factor to factor sums. <br> - Define exponential notation. <br> - Explore the meaning of a square number. <br> - Use order of operations conventions to simplify expressions. | 18 |
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## WORD BANK

| Word or Phrase | Definition or Description | Example or Picture |
| :--- | :--- | :--- |
| base |  |  |
| composite <br> number |  |  |
| exponent |  |  |
| factor |  |  |
| greatest common |  |  |
| factor |  |  |
| seast common |  |  |
| multiple |  |  |
| prime number |  |  |
| multiple |  |  |
| order of |  |  |
| operations |  |  |

## FACTORS AND MULTIPLES

## Summary

We will review factors and multiples of numbers, and prime and composite numbers.

## Goals

- Understand what it means to find factors and multiples of numbers.
- Review the definitions of prime and composite numbers.

In this lesson, we will only work with natural numbers. The natural numbers are the numbers

## $1,2,3, \ldots$. Natural numbers are also referred to as counting numbers.

List the next nine multiples of each given number.

1. 2 , $\qquad$
$\qquad$
$\qquad$ —— $\qquad$ , $\qquad$ , $\qquad$ ,
2. 5, $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$
3. 

7, $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$
4. 8 , $\qquad$ , $\qquad$ , $\qquad$ _, $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$
5. 9 , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ ,
6. Describe in your own words what it means for a number to be a multiple of another number.
7. Do some numbers have more multiples than other numbers? Explain.

## BUILDING RECTANGLES

Each square in the rectangles to the right represents one square unit of area. These rectangles, which are drawn on a grid, both have an area of 6 square units. We will call these "grid rectangles" and label their dimensions as $3 \times 2$ and $2 \times 3$.

1. Give the dimensions of the two other grid rectangles whose areas
 are 6 square units.
2. Build and draw all possible grid rectangles with an area of 12 square units. Use space efficiently when recording below.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  | \| |  |  |  |  |  |  |  |  |  |  |  |  |

3. What are all the possible dimensions of grid rectangles with area of 12 square units (written as pairs like $2 \times 3$ above)?
4. Draw all grid rectangles with area equal to 7 square units, 9 square units, and 1 square unit. Then complete the table. Use additional grid paper if necessary.

| area of rectangle | Dimensions of the rectangles |
| :--- | :--- |
| a. 7 square units |  |
| b. 9 square units |  |
| c. 1 square unit |  |

## BUILDING RECTANGLES (Continued)

5. Describe in your own words what it means for a number to be a factor of another number.
6. List all of the factors of the following in pairs (for example, the factors of 6 in pairs are 1 and 6 , and 2 and 3 )

## $12 \rightarrow$

$9 \rightarrow$
7. What is the relationship between the dimensions of the grid rectangles on the previous page and the factors of the numbers that describe their areas?
8. How many different grid rectangles can be drawn with 12 square units? $\qquad$ with 7 square units? $\qquad$ with 9 square units? $\qquad$ with 1 square unit? $\qquad$
9. Without actually drawing them, how many grid rectangles can be drawn with 10 square units? List their dimensions in pairs to justify your answer.
10. Without drawing, how many grid rectangles can be drawn with 18 square units? Justify by listing their dimensions in pairs.
11. Why do you think 9 is called a square number, while 7 and 12 are not?
12. Is 1 a square number? Explain.
13. List at least four more square numbers. Explain how you know they are square numbers.

## BUILDING RECTANGLES (Continued)

A prime number is a natural number that has exactly two unique factors, namely 1 and itself.
A natural number is composite if it has more than two factors.
14. Classify each number below as prime, composite, or neither, and explain how you know.

| $12 \rightarrow$ | $7 \rightarrow$ |
| :--- | :--- |
| $9 \rightarrow$ | $1 \rightarrow$ |

15. List all of the prime numbers less than 20.
16. List all of the composite numbers less than 20.
17. Explain why all square numbers greater than 1 must be composite.
18. Explain why 2 can be the only even prime number.

The fundamental theorem of arithmetic states that every natural number greater than 1 is either prime or the product of prime factors.

Example: 24 is not prime, and can be written as the product of primes:

$$
24=2 \cdot 2 \cdot 2 \cdot 3
$$

19. Write 28 as a product of prime factors.

## PRIME FACTORIZATION: FACTOR TREES

A "factor tree" is a useful organizational tool for writing numbers as a product of prime factors (also called finding the prime factorization of a number).

- Write any pair of factors of the given number (except 1 and itself).
- If a number cannot be factored further (in other words, if it is prime), circle it.
- Factor all numbers until none can be factored. The product of the circled prime numbers is the original number.

1. Find the prime factorization of 36 in three different ways with the given head starts.

$36=$ $\qquad$
Find the prime factorization of each using a factor tree. Only one way is required.

| 2.20 | 3. | 42 | 66 |
| :--- | :--- | :--- | :--- |

## FACTORS CHART

Complete this chart by following the directions of your teacher.

FACTORS OF THE NUMBERS

| 50 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 49 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 48 | 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 47 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 46 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 45 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 44 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 43 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 42 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 41 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 40 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 39 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 38 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 37 | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 36 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 35 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 34 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 33 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 32 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 31 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 30 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 29 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 28 | 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 27 | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 26 | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 25 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 24 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 23 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 22 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 21 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 20 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 19 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 18 | 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 17 | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 16 | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 15 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 14 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 6 |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 3 |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 | 2 |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1 | 23 | 34 | 45 | 6 | 7 | 8 | 910 | 1011 | 112 | 1213 | 14 | 1516 | 1617 | 718 | 119 | 20 | 2122 | 22.23 | 324 | 25 | 2622 | 2728 | 829 | 30 | 31 | 32 | 33 | 34 | 35 | 363 | 738 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 4647 | 4748 | 849 | 50 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | N | UM | AB | ER | RS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## EXPLORING PATTERNS OF FACTORS

1. Describe some patterns that you observe in the chart on page 6.
2. Complete the table.

Numbers that have

| exactly: |
| :---: |
| one factor |
| two factors |

## A MULTIPLICATION CHART

1. Complete the multiplication chart.

| 1 | 2 | 3 |  |  |  | 7 | 8 | 9 |  | 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 |  |  |  |  |  |  |  | 20 |  |  |
|  |  | 9 |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  | 24 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  |
|  | 14 |  |  | 35 |  |  |  |  | 70 |  |  |
|  |  | 24 |  |  |  |  |  |  | 80 |  | 96 |
|  |  |  | 36 | 45 |  |  |  |  |  |  | 108 |
| 10 |  |  |  |  |  | 70 |  |  |  |  |  |
| 11 |  |  |  |  | 66 |  | 88 | 99 |  |  |  |
| 12 | 24 |  |  |  |  | 84 |  |  |  |  |  |

2. Use $7 \cdot 12$ as an example to explain how to use the multiplication chart to find products.
3. Use $24 \div 6$ as an example to explain how to use the multiplication chart to find quotients.

## A MULTIPLICATION CHART (Continued)

4. Explain how to use the multiplication chart to list multiples of a number.
5. Explain how to use the multiplication chart to find squares of a number and describe where the square numbers are located.
6. The multiplication chart can be used to identify equivalent fractions. Look at rows 1 and 2 only. How can you use these rows to list fractions that are equivalent to $\frac{1}{2}$ ?
7. List at least five fractions from rows 1 and 2 that are equivalent to $\frac{1}{2}$.
8. Use rows 2 and 5 to find and list at least five fractions that are equivalent to $\frac{2}{5}$.
9. A multiplication chart can be used to find common multiples of two numbers. List four multiples that 6 and 9 have in common.
10. Ari says that he can tell that 108 is a multiple of 6 by using the chart. Is Ari correct? Explain.

## GCF AND LCM

## Summary

We will learn how to find the greatest common factor (GCF) of two natural numbers and the least common multiple (LCM) of two natural numbers. We will use GCF and LCM to solve problems.

## Goals

- Find the greatest common factor of two natural numbers.
- Find the least common multiple of two natural numbers.
- Use the greatest common factor and least common multiple to solve problems.


## Warmup

1. Write the first six multiples of 11.
2. Write all factors of 30 in pairs.
3. Explain why 84 is a composite number.
4. Find the prime factorization of 80 .

## GREATEST COMMON FACTOR

1. A factor of a natural number is always $\qquad$ the number itself.
less then, less than or equal to, greater than, greater than or equal to
To begin, let's make sense of the phrase "greatest common factor" (GCF) by using words and the numbers 12 and 18.
2. List all the factors of 12 : $\qquad$
3. List all the factors of 18 : $\qquad$
4. In problems 2 and 3 , circle all the factors that 12 and 18 have in common.
5. Look at the circled numbers. The greatest factor that 12 and 18 have in common is $\qquad$ .
6. Describe in your own words why the number you wrote for problem 5 is the greatest common factor of 12 and 18.

Use the process described above to find the GCF of each pair of numbers.

| 7. 10 and 15 | 8.3 and 8 |  |
| :--- | :--- | :--- |
| 9. | 14 and 42 | 10.36 and 48 |
|  |  |  |

## LEAST COMMON MULTIPLE

1. A multiple of a natural number is always $\qquad$ the number itself.
less then, less than or equal to, greater than, greater than or equal to

To begin, let's make sense of the phrase "least common multiple" (LCM) by using words and the numbers 4 and 6 .
2. List the first ten multiples of 4 : $\qquad$
3. List the first ten multiples of 6 : $\qquad$
4. Circle all the multiples that 4 and 6 have in common.
5. Look at the circled numbers. The least multiple that 4 and 6 have in common is $\qquad$ .
6. Describe in your own words why the number you wrote for problem 5 is the least common multiple of 4 and 6.

List several multiples of each number to find the LCM of each pair of numbers.

| 7. 6 and 12 | 8. | 3 and 7 |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| 9. | 8 and 10 | 10. | 5 and 6 |

## SOLVING PROBLEMS

Solve each problem and explain your solution using multiple representations.

1. Kristi has 16 apple slices and 24 grapes. If she wants to make identical snack packs using all of the fruit, what is the largest number of packs can she make?
2. Karen exercises at the gym every 4 days and David exercises at the gym every 10 days. Karen and David both exercised today. How many days from today will be the next day they exercise together?
3. For which problem is the GCF the solution? Explain why.
4. For which problem is the LCM the solution? Explain why.

## GCF AND LCM SHORTCUTS

Shelley must find the GCF and LCM of 12 and 18. She likes to find GCF's and LCM's using the following procedure.

1. Make factor trees for 12 and 18 and write each number as a product of primes.

$12=$ $\qquad$ $18=$ $\qquad$
2. Write each factor that 12 and 18 share in the overlapping part of the o .

Write other factors of each number in the nonoverlapping part of each circle.

3. The product of the prime factors in the overlap is $\qquad$ . This is the $\qquad$ of 12 and 18.
4. The product of the factors in the diagram is $\qquad$ . This is the $\qquad$ of 12 and 18.
5. Use factor trees and sorting circles to find the GCF and LCM for 8 and 12.

## GCF AND LCM SHORTCUTS (Continued)

Mark must find the GCF and LCM of 16 and 12. He likes to find GCF's and LCM's using an "upside down ladder" procedure.
6. Follow the steps below and fill in the blanks with the appropriate numbers.


- Divide each number by any common factor greater than 1. Notice that 2 is used as the common factor in the first line.
- Write the quotients below their respective dividends.

Notice that $16 \div 2=8$ and $12 \div 2=6$

- Continue until both resulting quotients have no factors in common greater than 1.

7. The product of the common factors along the side is $\qquad$ . This is the $\qquad$ of 16 and 12.
8. The product of the factors along the side together with the quotients along the bottom is . This is the of 16 and 12 .

Use this process to find the GCF and LCM of the following.

| 9. 8 and 10 | 10. | 32 and 40 |  |
| :--- | :--- | :--- | :--- |
| GCF: |  |  |  |
|  | LCM: |  |  |

## MORE GCF AND LCM

Find the GCF and LCM of each pair of numbers. Use any method.


For each problem, identify if GCF or LCM could be used to solve it. Then solve each problem.
4. Megan has two pieces of cloth. One piece is 45 inches wide. The other is 60 inches wide. She wants to cut strips of equal width that are as wide as possible with no fabric left over. How wide should she cut the strips?
5. Stacey has 8 -inch pieces of toy train track and Paul has 18 -inch pieces of train track. They want to build the shortest possible tracks that are the same length. How long would the track be? How many pieces does Stacey need and how many does Paul need?
6. Mrs. Stern has 120 markers and 30 pieces of paper to give to her students. What is the largest number of students she can have in her class so that each student gets equal numbers of markers and equal number of pieces of paper?

## PRACTICE

1. List all the factors of 48 in pairs. $\qquad$
2. Describe in words what it means for a number to be a factor of another number.
3. List the first seven multiples of 9 . $\qquad$
4. What are prime numbers?
5. What are composite numbers?
6. What natural number has exactly one factor? $\qquad$
7. List the first eight prime numbers. $\qquad$
8. List the first eight composite numbers.
9. List the first eight square numbers.
s.
10. Find the prime factorization of 550.

For problems 11 and 12 , identify if GCF or LCM could be used to solve it. Then solve each problem.
11. In one hour, WLCM radio gave away one $\$ 50$ bill to every $50^{\text {th }}$ caller and two free movie tickets to every $20^{\text {th }}$ caller. How many callers got through before one of them received both a $\$ 50$ bill and two free movie tickets?

If 224 people called WLCM radio station in one hour, how much money did they give away? How many movie tickets did they give away?
12. Zack is making a game board that is 16 inches by 24 inches. He wants to use square tiles. What is the largest tile he can use? How many tiles does he need?

## NUMERICAL EXPRESSIONS

## Summary

We will use our knowledge of greatest common factor to write equivalent expressions using the distributive property. We will learn about exponents. We will simplify expressions using the conventions for the order of operations.

## Goals

- Apply the concept of greatest common factor to factor sums.
- Define exponential notation.
- Explore the meaning of a square number.
- Use order of operations conventions to simplify expressions.

Find GCF and LCM of each of the following pairs of natural numbers. Use any method.

| 7. 8 and 10 | 8. | 2 and 3 |
| :--- | :--- | :--- |
|  |  |  |
| 9. | 40 and 50 | 10. |

11. When the GCF of a pair of numbers is 1 , we refer to them as relatively prime. Relatively prime numbers need not be prime. What are the relatively prime pairs of numbers above?
12. Make a conjecture about the LCM of relatively prime pairs of numbers.

## GCF AND THE DISTRIBUTIVE PROPERTY

The distributive property states that $a(b+c)=a b+a c$ and $(b+c) a=b a+c a$ for any three numbers $a, b$, and $c$. This property relates two operations (multiplication and addition). It is called the "distributive property" because it "distributes" the factor outside the parentheses over the two terms within the parentheses.

Examples: $3(5+4)=3(5)+3(4) ; \quad(4+5) 8=4(8)+5(8) ; \quad 6(4-1)=6(4)-6(1)$

Use the distributive property to rewrite the expressions like in the examples above.


Verify that each of these expressions above is equivalent.
5. Recall a shortcut from the previous lesson for finding GCF (started at the right)
a. Finish the process.

b. What is the GCF of 16 and 20 ? $\qquad$
c. Use the GCF to rewrite $16+20$ as a product, as in problem 4 above.

## GCF AND THE DISTRIBUTIVE PROPERTY (Continued)

6. Circle all of the equations below that correctly illustrate the distributive property.
a. $10+25=5(2+5)$
b. $\quad 9+24=3(3+24)$
c. $\quad 5(6+2)=5(6)+5(2)$
d. $\quad 40-16=8(5-2)$

On the previous page we used the distributive property to express the sum of two numbers as the product of their GCF and the sum of two whole numbers with no common factors.

For each problem below, write each sum as a product. Then verify that the given expression and the resulting expression are equivalent. We call this process "factoring."

Example:
Check: $\quad 18+24=42$
$=6(3)+6(4)$
$=6(3+4)$

$$
6(3+4)=6(7)=42
$$

7. $14+21=$
8. $15+25=$ $\qquad$
Check:
Check:
9. $24-8=$ $\qquad$
Check:
Check:

Factor these sums.

| 11. $10+20+30=$ |  |
| :--- | :--- |
| Check: | 12. $2+4+6+8=$ <br> Check:  |

## EXPONENTS AND AREA

The exponential notation $b^{n}$ (read as " $b$ to the power $n$ ") is used to express $n$ factors of $b$. The number $b$ is the base, and the natural number $n$ is the exponent.

Example: $\quad 2^{3}=2 \cdot 2 \cdot 2=8 \quad$ The base is 2 and the exponent is 3 .

Write each expression as an appropriate product. Then compute.

1. $5^{2}=$ $\qquad$ - $\qquad$ $=$ $\qquad$
2. $3^{4}=$ $\qquad$ $=$ $\qquad$
3. $2^{6}=$ $\qquad$ $=$
4. $17^{1}=$ $\qquad$ $=$ $\qquad$
5. When doing the example problem in the box above, Horton wrote $2^{3}=6$. Explain Horton's mistaken thinking.

Compute.

| 6. $3^{2}$ | 7. $2^{5}$ | 8. $4^{3}$ |
| :---: | :---: | :---: |
| 9. $6^{2}$ | 10. $3^{2}+3^{4}$ | 11. $2^{3} \cdot 3^{2}$ |
| 12. $2^{5}+2^{5}$ | 13. $2\left(2^{5}\right)$ | 14. $3\left(3^{1}+3^{2}\right)$ |

## EXPONENTS AND AREA (Continued)

You have learned before that to find the area of a rectangle, you multiply length times width. Let's review why this makes sense.
15. Each small square in the rectangles below are 1 square unit of area.
a. What is the length of each side? $\qquad$ and $\qquad$
b. How many rows of 3 are there? $\qquad$

c. What is $2 \times 3$ ? $\qquad$
d. How many columns of 2 are there?
e. What is $3 \times 2$ ? $\qquad$
f. How many square units are in this rectangle? $\qquad$
16. Recall that a square is simply a special rectangle with sides of the same length.
a. What is the length of each side? $\qquad$

c. How many columns of 4 are there? $\qquad$
d. What is $4 \times 4$ ? $\qquad$
e. How many square units are in this square? $\qquad$
f. Rewrite the expression $4 \times 4$ using a base number and an exponent. $\qquad$
17. Here is another square and rectangle.
a. What is the area of the square? $\qquad$

b. What is the area of rectangle? $\qquad$
c. What is the total area of both combined? $\qquad$

d. Write at least two different expressions that represent how you could find this total area. Try to write at least one expression that includes an exponent.

## ORDER OF OPERATIONS

The order in which we perform mathematical calculations is determined by agreed-upon rules. Order of operations is a mathematical convention.

## Order of Operations

1. Simplify expressions that are grouped.
2. Compute expressions with exponents.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction from left to right.

| Evaluate each expression | List the operations in order from first to last |
| :---: | :---: |
| 1. $72 \div(1+5)^{2}+4 \cdot 3$ | 1. Grouping (add $1+5$ in parenthesis) <br> 2. Exponent (compute 6 to the $2^{\text {nd }}$ power) <br> 3a. Multiplication and division from left to right (divide $72 \div 36$ ) <br> 3b. Multiplication and division from left to right (multiply $4 \cdot 3$ ) <br> 4. Addition and subtraction from left to right |
| 2. $16 \div 8 \cdot 2^{3}$ |  |
| 3. $24 \div 6+2 \cdot 3$ |  |
| 4. $24 \div(6+2) \cdot 3$ |  |

## ORDER OF OPERATIONS (Continued)

| Evaluate each expression | List the operations in order from first to last |
| :--- | :--- | :--- |
| 5. $\frac{12+8}{4-2}$ |  |
|  |  |
| 6. $12+8 \div 4-2$ |  |
| 7. $12+8) \div 4-2$ |  |

Simplify each expression.

| 8. | $6^{2}-12 \div 6 \div 2$ | 9. | $\frac{6^{2}-12 \div 6}{2}$ | 10. $\frac{6^{2}-12}{6 \div 2}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 11. | $(36-8) \div 14+6 \div 2$ | 12. | $(10+2) \div 4$ | 13. | $10+2 \div 4$ |

## ORDER OF OPERATIONS CHALLENGES

Place parentheses in the equations below so that each becomes a true statement. Use as many sets of parentheses as needed to make your work clear. Write "none needed" if the equation is already true.

| 1a. $2+4 \cdot 6 \div 3=10$ <br> 2a. $1+3^{2} \cdot 2+5=25$ <br> 1b. $2+4 \cdot 6 \div 3=12$ <br> 3. Alexandra says that both sets of parentheses for the problem below are necessary to <br> make the equation true. Is Alexandra correct? Explain. <br> 3 + (5 - 2) $\div(7-5)=8$ 2b. $1+3^{2} \cdot 2+5=70$ |
| :--- |

Use 8, 6, and 2 exactly once each, in any order, to create an expression with each indicated value. You may use any operation symbols and grouping symbols you like.


## SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1

1. For the number $7,157,823$, name the two places that include the digit 7 .
2. For the number 157.56, name the two places that include the digit 5 .

Find the value of each expression and write the property that each illustrates .
3. $(9+25)+5=9+(25+5)$
4. $5(17+23)=5(17)+5(23)$

Property:
Property: $\qquad$

Explain which expression above was easier for you to compute.

Explain which expression above was easier for you to compute.
5. Compute. Then rewrite the original subtraction problem as a subtraction expression.

5,613
-921
6. Compute. Then rewrite the original addition problem, including the sum you find, as an addition equation.

8,437
604
219
7. Create a word problem that could answered with the calculation in problem 6.

## SKILL BUILDER 2

Find each product.
1.
a. $5 \times 11$
b. $\quad 50 \times 11$
c. $110 \times 50$
d. $\quad 1,100 \times 50$
2.
a. $\quad 12 \times 4$
b. $\quad 40 \times 12$
c. $\quad 40 \times 120$
d. $\quad 120 \times 4,000$

Find two different ways to derive each multiplication fact.

| 3. $7 \times 8$ |  |  |
| :--- | :--- | :--- |
| Method 1: | Method 2: | 4. $12 \times 8$ |
| Method 1: |  |  |

5. For the number 392.83, name the two places that include the digit 3.
6. Multiply 510 by 19 using an area model. Check your answer using another method.

| Solution: | Check: |
| :--- | :--- |
|  |  |

7. Find the value of points $A$ and $B$ on the number line. All marks on the line are equally spaced. Explain how you found your answers.


## SKILL BUILDER 3

1. Use the alternative division algorithm. Check your answer using an area model for multiplication.

2. There are 384 sixth grade students going on a field trip. A bus can hold 58 people.
a. How many buses are needed to hold all of the students?
b. Will your answer to this question change if we now know that 10 adults have to fit on the buses as well? Explain.

## SKILL BUILDER 4

1. Circle all the prime numbers in the list below.

| 12 | 13 | 5 | 21 | 17 | 2 | 9 | 18 | 25 | 33 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

2. List all the prime numbers that are greater than 40 and less than 60 .
3. List all the composite numbers that are greater than 20 and less than 40 .
4. Explain why 1 is the only natural number that is neither prime nor composite.

Find the prime factorization of each number using a factor tree. Only one way is required. Write your result as an equation.

| 5. | 64 | 64 | 7. | 96 |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

8. Ramon incorrectly thinks that the prime factors for 30 are 3 and 10 . Write an explanation to Ramon that explains his error.

## SKILL BUILDER 5

1. List all the factors of 30 . $\qquad$
2. List all the factors of 36 . $\qquad$
3. Circle all the factors that 30 and 36 have in common.
4. What is the greatest factor that 36 and 30 have in common?
5. Describe in your own words why the number you wrote for problem 4 is the greatest common factor of 36 and 30 .

Use the process described above to find the GCF of each pair of numbers.

| 6. 72 and 50 | 7. | 100 and 150 |
| :--- | :--- | :--- |
| 8. 34 and 68 | 9. |  |

10. Explain why 75 and 14 do not have a common factor greater than 1 .
11. Besides 75 and 14 , find another pair of numbers that do not share a common factor greater than 1. Explain your reasoning.
12. List the first ten multiples of 8 . $\qquad$
13. Circle all the multiples that 8 and 12 have in common.
14. What is the least multiple that 8 and 12 have in common?
15. Describe in your own words why the number you wrote for problem 4 is the least common multiple of 8 and 12.

List several multiples of each number to find the LCM of each pair of numbers.

| 6. 5 and 7 | 7. 4 and 10 |  |
| :--- | :--- | :--- |
| 8. | 12 and 18 | $9 . \quad 10$ and 6 |
|  |  |  |

10. Sadie is trying to find the LCM for 9 and 6 . She thinks the LCM is 36 because both 9 and 6 divide evenly into 36.
a. Critique Sadie's reasoning.
b. What might help Sadie to avoid making this mistake in the future?

## SKILL BUILDER 7

Find the GCF and LCM of each pair of numbers. Use any method.


For each problem, identify if GCF or LCM could be used to solve it. Then solve each problem.
4. During a holiday sale, a large company was giving out $\$ 10$ gift cards to every 40th customer and a coupon to every 15 th customer. How many customers checked out before one customer got both the gift card and the coupon?
5. Janet wants to use square tiles to cover the floor of her new shower. The shower is 45 inches by 55 inches. What's the largest size of square tiles that Janet could use if she wanted to use whole tiles only?
6. Mr. Rogers has 75 erasers and 50 pencils to give to his students.
a. What's the largest number of students that he can have in his class so that each student gets the same number of erasers and pencils assuming he gives away all his pencils and erasers?
b. If he had 15 students in his class, could he divide them evenly among his students assuming he gives away all his pencils and erasers? Explain.

## SKILL BUILDER 8

For each problem in the table below write each sum as a product. Then verify that the given expression and the resulting expression are equivalent.

| $\text { Example: } \quad \begin{aligned} 18+24 & =6(3)+6(4) \\ & =6(3+4) \end{aligned}$ | Check: $18+24=42$ <br> and $6(3+4)=6(7)=42$ |
| :---: | :---: |
| 1. $12+18$ | Check: |
| 2. $20+30$ | Check: |
| 3. $18+6$ | Check: |
| 4. $4+12$ | Check: |

Compute.

| 5. | $5^{2}$ | 6. | $2^{3}$ | 7. | $3^{3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 8. | $9^{2}$ | 9. | $2^{2}+3^{4}$ |  |  |

## FOCUS ON VOCABULARY



## Across

28 8, 24, and 14 are examples of $\qquad$ numbers

5 Agreed-upon conventions, such as order of operations

7 A number greater than 1 that is not composite

9 $\qquad$ (counting) numbers

Diagram (sorting circles)

11 A branching diagram used to find factors of numbers

12 A shape whose area is length $\times$ width

Down
16 is the $\qquad$ of 12 and 18
(abbreviation)
324 is a $\qquad$ of 8
$41,4,9,16$, and 25 are the first 5 $\qquad$ numbers

6 The " 3 " in the expression $5^{3}$
$8 \quad 8$ is a $\qquad$ of 24

1324 is the $\qquad$ of 8 and 12 (abbreviation)

## SELECTED RESPONSE

Show your work on a separate sheet of paper.

1. Choose all the numbers that are prime.
A. 2
B. 9
C. 1
D. 21
E. 13
F. 57
2. Choose all the expressions that are the prime factorization of 24 .
A. $4 \cdot 6$
B. $3 \cdot 8$
C. $2 \cdot 2 \cdot 2 \cdot 3$
D. $2^{3} \cdot 3$
3. Choose all statements that are FALSE about the numbers 12 and 8.
A. Both numbers are composite numbers
C. The LCM of the numbers is 72 .
E. The GCF of the numbers is 4 .
B. Both numbers are prime numbers.
D. The LCM of the numbers is 4 .
F. The LCM of the numbers is 24 .
4. Which of the following statements are TRUE about the number 3 and 7 . Choose all that apply.
A. Both numbers are composite numbers
B. Both numbers are prime numbers.
C. The LCM of both numbers is 21 .
D. The LCM of both numbers is 42 .
E. The GCF of both numbers is 21 .
F. They do not share a common factor.
5. Circle all of the equations below that correctly represent using the distributive property to rewrite a sum as a product.
A. $12+18=6(2+3)$
B. $10+15=25$
C. $14+12=2 \cdot 13$
D. $6+8=2(2+5)$
6. Choose all of the expressions that have a value of 16 .
A. $2+2^{2}$
B. $(2+1)^{3}-10+1$
C. $4+2 \cdot 2$
D. $8 \cdot 6 \div(2+1)$

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.
2.1 Factors and Multiples

1. List all the prime numbers greater than 60 and less than 70 .
2. List all the composite numbers that are greater than 20 and less than 30 .
3. Create a prime factorization tree for 54 .

### 2.2 GCF and LCM

4. Find the GCF and LCM for the following numbers.
a. 15,10
b. 14,35

GCF: $\qquad$ LCM: $\qquad$ GCF: LCF: $\qquad$

### 2.3 Numerical Expressions

5. Evaluate the expression below and then list the operations in order from first to last. $4+(5-2)^{2} \cdot 5$
6. Write the sum as a product. Then verify that the given expression and the resulting expression are equivalent.
$12+14$
Check:

## HOME SCHOOL CONNECTION

1. Make a prime factorization tree for the following numbers. Write the prime factorizations as equations.
a. 8
b. 13
c. 42
d. 64
2. Write a few sentences to explain the difference between the LCM of two numbers and the GCF of two numbers. Support your explanation with numerical examples.
3. Explain why the following expressions are not equivalent, even though they look very similar.
a. $\quad 10+3+2^{2} \cdot 3$
b. $\quad 10+(3+2)^{2} \cdot 3$
$\qquad$

## COMMON CORE STATE STANDARDS - MATHEMATICS

| STANDARDS FOR MATHEMATICAL CONTENT |  |  |
| :---: | :---: | :---: |
| 6.NS. 4 | Find the greatest common factor of two who common multiple of two whole numbers less express a sum of two whole numbers $1-10$ whole numbers with no common factor. For | hole numbers less than or equal to 100 and the least ss than or equal to 12 . Use the distributive property to 00 with a common factor as a multiple of a sum of two r example, express $36+8$ as $4(9+2)$. |
| 6.EE. 1 | Write and evaluate numerical expressions | involving whole-number exponents. |
| 6.EE.2b | Write, read, and evaluate expressions in w expression using mathematical terms (sum or more parts of an expression as a single as a product of two factors; view $(8+7)$ as | which letters stand for numbers: Identify parts of an n , term, product, factor, quotient, coefficient); view one entity. For example, describe the expression 2(8+7) both a single entity and a sum of two terms. |
| STANDARDS FOR MATHEMATICAL PRACTICE |  |  |
|  |  |  |
| MP1 | Make sense of problems and persevere in solving them. |  |
| MP2 | Reason abstractly and quantitatively. |  |
| MP3 | Construct viable arguments and critique the reasoning of others. |  |
| MP5 | Use appropriate tools strategically. |  |
| MP6 | Attend to precision. |  |
| MP7 | Look for and make us of structure. |  |
| MP8 | for and express regularity in repeated reas | soning |

